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Dr. Kamlesh Kumar
Dept. of Mathematics

B.Sc. Part II , Paper-III

(Group-B) INFINITE SERIES

If $\sum u_n$ and $\sum v_n$ be two series of positive terms then $\lim_{n\to\infty} \frac{u_n}{v_n}$ be finite and non-zero, the series will be both convergent or divergent.

The infinite series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots$ to ∞ is convergent if $p>1$ and divergent if $p \le 1$.

Rule for Solving :-

(i) To find the nth term u_n.

(ii) To consider another auxiliary series whose nth term (v_n) is equal to

$$v_n = \frac{\text{Term of the highest power of } n \text{ in numerator of } u_n}{\text{Term of highest power of } n \text{ in denominator of } u_n.}$$

(iii) To find $\frac{u_n}{v_n}$.

(iv) To find $\lim_{n\to\infty} \frac{u_n}{v_n}$. If $\lim_{n\to\infty} \frac{u_n}{v_n}$ = finite and nonzero then proceed as following.

(v) Compare v_n with the series $\sum \frac{1}{n^p}$

$\sum \frac{1}{n^p}$ is convergent if $p>1$ and divergent if $p \le 1$.

(vi) By comparison test both the series $\sum u_n$ and $\sum v_n$ will converge or diverge simultaneously

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Ex 1. → Test the convergence of the series

$$1+\frac{1+2}{1+2^2}+\frac{1+3}{1+3^2}+\cdots+\frac{1+n}{1+n^2}+\cdots \text{ to } \infty$$

Soln → Given series also written as

$$\frac{1+1}{1+1^2}+\frac{1+2}{1+2^2}+\frac{1+3}{1+3^2}+\cdots+\frac{1+n}{1+n^2}+\cdots \text{ to } \infty$$

Then its n^{th} term $U_n=\frac{1+n}{1+n^2}$

Let us consider another series whose n^{th} term is V_n and $V_n=\frac{n}{n^2}=\frac{1}{n}$

$$\therefore \frac{U_n}{V_n}=\frac{\frac{1+n}{1+n^2}}{\frac{1}{n}}=\frac{n(1+n)}{1+n^2}=\frac{n^2+n}{1+n^2}=\frac{n^2(1+\frac{1}{n})}{n^2(\frac{1}{n^2}+1)}$$

$$=\frac{1+\frac{1}{n}}{1+\frac{1}{n^2}}$$

$\therefore \lim_{n\to\infty}\frac{U_n}{V_n}=\lim_{n\to\infty}\frac{1+\frac{1}{n}}{1+\frac{1}{n^2}}=1$, which is finite and non-zero.

So, by comparison test, the two series ΣU_n and ΣV_n shows alike (i.e. converge or diverge simultaneously)

Now, $V_n=\frac{1}{n} \Rightarrow \Sigma V_n=\Sigma\frac{1}{n}=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}+\cdots$ to ∞

Comparing it with $\Sigma\frac{1}{n^p}$, we get $p=1$

$\Rightarrow \Sigma V_n$ is not convergent that means divergent

Hence, by comparision test the given series ΣU_n is divergent.

Ex 2 → Test the convergency of the series $\Sigma\frac{5n-3}{2n^3-1}$

Soln – Here $U_n=\frac{5n-3}{2n^3-1}$, consider another series $\Sigma V_n=\frac{n}{n^3}=\frac{1}{n^2}$

$$\therefore \frac{U_n}{V_n}=\frac{n^2(5n-3)}{2n^3-1}=\frac{\frac{n^2(5n-3)}{n^3}}{\frac{(2n^3-1)}{n^3}}=\frac{5-\frac{3}{n}}{2-\frac{1}{n^3}}$$

$\therefore \lim_{n\to\infty}\frac{U_n}{V_n}=\lim_{n\to\infty}\frac{5-\frac{3}{n}}{2-\frac{1}{n^3}}=\frac{5}{2}$, which is finite and non-zero.

So, by comparision test, they behave alike

But $\Sigma V_n=\Sigma\frac{1}{n^2}=\frac{1}{1^2}+\frac{1}{2^2}+\cdots+\frac{1}{n^2}+\cdots$ to ∞ is convergent.

$\Rightarrow \Sigma U_n$ (the given series) is also convergent